Warm-up!

For each of the follow, determine what degree polynomial would best fit the points. Justify your choices by completing the sentence "Consecutive equally spaced input values correspond with…"

2

x	-2	1	4	7	10	13
$f(x)$	15	8	4	3	5	10

-7 -4 -1 +2 +5

+3 +3 +3 +3

1

x	-10	-8	-6	-4	-2	0
$g(x)$	-14	-11	-8	-5	-2	1

+3 +3 +3 +3 +3

x	3	4	5	6	7	8
$h(x)$	-7	-6	-4	0	8	24

+1 +2 -4 +8 +16

3

x	-10	-5	0	5	10	15
$k(x)$	9	10	7	6	13	34

+1 -3 -1 +7 +21

-4 +2 +8 +14

+6 +6 +6

Exponential Functions Day 2:

- Linear and Exponential Functions
- Exponential Function Graphs
- Polar Conics

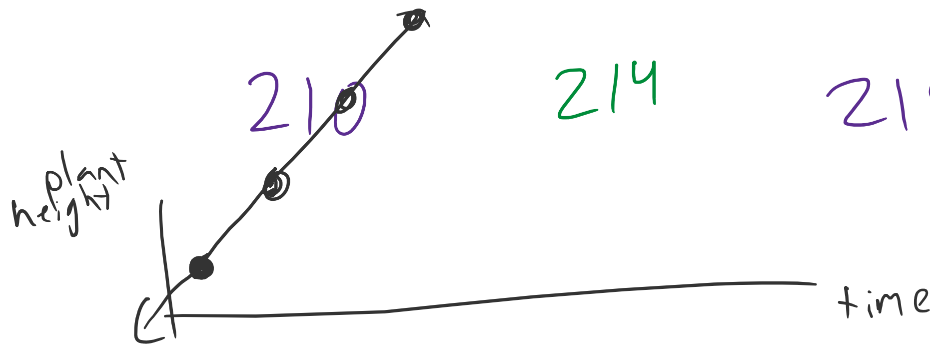
Homework: Linear and Exponential Functions

Your next quiz will be Day 5 of this unit

Should the below situation be modelled by an arithmetic sequence, a geometric sequence, or neither?

Violet is growing an aloe plant. Every morning, she measures its height. She finds that each day the height grows by 8 millimeters. For example, yesterday at 7:40 AM it was 202 mm tall and this morning at 7:40 AM it was 210 mm tall.

yesterday 7:40 AM	today 7:40 AM	today 7:40 PM	tomorrow 7:40 AM
202	210	214	218



Linear Functions are Arithmetic Sequences with a Broader Domain

Arithmetic Sequence	Linear Functions
$a_n = a_k + d(n - k)$	$y = y_1 + m(x - x_1)$
$a_n = a_0 + dn$	$y = b + mx$

Over equal-length input-value intervals, the output values change at a constant rate.

The change in output is based on adding.

These two questions are essentially the same

Willy collects inflatable Christmas decorations. Each December he purchases the same number of decorations. If he has 32 inflatables at the end of his 10th year doing this and has 44 inflatables at the end of his 14th year doing this, predict how many inflatables Willy will have at the end of his 30th year doing this.

$a_{10} = 32$ $a_{14} = 44$ $a_{30} = ?$

$a_{14} = a_{10} + 4d$

$44 = 32 + 4d$

$12 = 4d$

$d = 3$

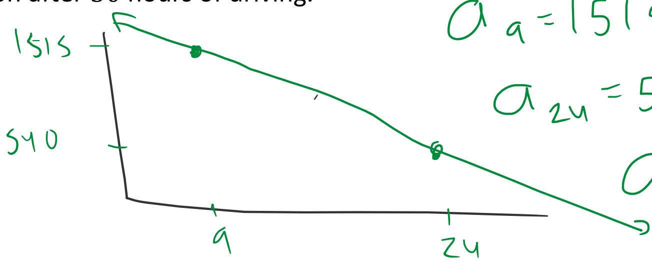
$a_{30} = a_{14} + 16d$

$a_{30} = 44 + 16(3)$

$44 + 48$

$a_{30} = 92$

Veruca and Julie driving to Johnson City (Tennessee) at a constant speed. After 9 hours of driving, they are 1515 miles from their destination. After 24 hours of driving, they are 540 miles from their destination. Predict how far they are from her destination after 30 hours of driving.



$a_9 = 1515$

$a_{24} = 540$

$a_{30} = ?$

Which of these are linear functions?

x	-20	-10	0	10	20	30
$f(x)$	2	5	10	17	26	37

+3 +5 +7 +9 +11

+2 +2 +2

x	-4	-1	2	5	8	11
$g(x)$	5	12	19	26	33	40

+7 +7 +7 +7 +7

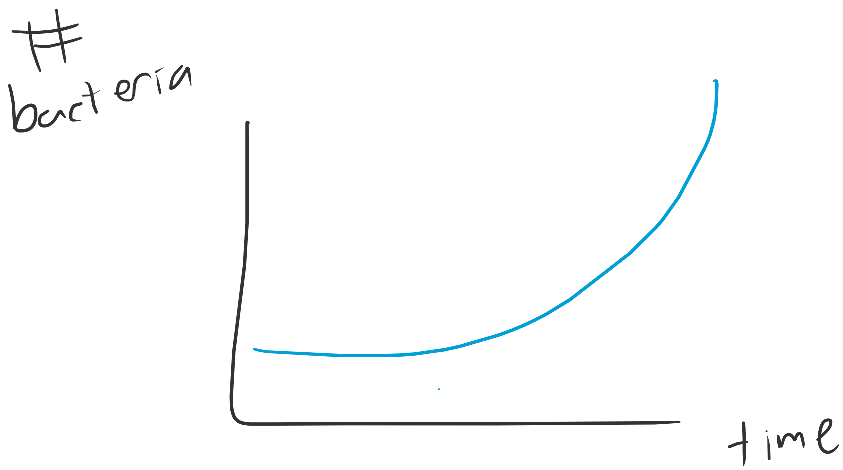
x	1	10	100	1000	10000	100000
$h(x)$	50	45	40	35	30	25

−5 −5 −5 −5 −5

Over equal-length input-value intervals, the output values change at a constant rate.

Should the below situation be modelled by an arithmetic sequence, a geometric sequence, or neither?

Charlie sneezes into a petri dish and then proceeds to every day measure how many bacteria are in the petri dish. Each day, the number of bacteria in the dish doubles.



Exponential Functions are Geometric Sequences with a Broader Domain

Geometric Sequence	Exponential Functions
$g_n = g_k(r)^{n-k}$	$y = y_1(b)^{x-x_1}$
$g_n = g_0(r)^n$	$y = a(b)^x$

Over equal-length input-value intervals, the output values change proportionally.

The change in output is based on multiplying.

These two questions are essentially the same

A super bouncy ball is dropped out of an airplane. On its first bounce, it reaches a height of 200 ft. On the fourth bounce, it reaches only 25 feet. If the height of the bounces form a geometric sequence, predict how the ball will reach on the 11th bounce.

$g_1 = 200$ $g_4 = 25$ $g_{11} = ?$

$g_4 = g_1 R^3$

$25 = 200 R^3$

$\frac{25}{200} = R^3$

$\frac{5}{40} = R^3$

$\frac{1}{8} = R^3$

$\frac{1}{2} = R$

$g_{11} = g_4 \cdot R^7$

$g_{11} = 25\left(\frac{1}{2}\right)^7$

$g_{11} = \frac{25}{128}$

Did you see the TikTok of the baby rapping? You haven't? How is that possible? I feel like everyone has seen it at this point. When the clock struck midnight on January 1st, it had only 200 view, but that number has been growing exponentially. By midnight of January 5th it had 16200 views. Predict how many views it has midnight of today. (you may use a calculator)

$g_1 = 200$ $g_5 = 16200$ $g_8 = ?$

$g_5 = g_1 R^4$

$16200 = 200 R^4$

$\frac{16200}{200} = R^4$

$81 = R^4$

$3 = R$

$g_8 = g_5 R^3$

$g_8 = 16200(3)^3$

$g_8 = 437400$

Which of these are exponential functions?

x	-20	-10	0	10	20	30
$f(x)$	22	19	13	1	-23	-71

-3 -6 -12 -24 -48

x	-4	-1	2	5	8	11
$g(x)$	-5	-10	-20	-40	-80	-160

-5 -10 -20 -40 -80

x	1	5	9	13	17	21
$h(x)$	162	108	72	48	32	$64/3$

-54 -36 -24 -16 $-\frac{32}{3}$

Over consecutive equal-length input-value intervals, the output values change proportionally or the first differences of the output values change proportionally.

Determine if the function is linear, exponential, or neither.

x	-6	-4	-2	0	2	4
$f(x)$	14	12	10	8	6	4

x	5	10	15	20	25	30
$g(x)$	193	112	58	22	-2	-18

Determine if the function is linear, exponential, or neither.

x	0	3	6	9	12	15
$h(x)$	-2	0	6	16	30	48

x	-9	-3	-1	1	3	9
$k(x)$	-800	-400	-200	-100	-50	-25

Information can spread exponentially/geometrically. This is great for true information but bad for rumors, misunderstandings, or disinformation.

(CALCULATOR) A piece of fake news is posted to Instagram. After 3 hours, 40 people believe the fake news. After 6 hours, 100 people believe the fake news. If the information is spreading exponentially, predict how many people will believe the fake news after 10 hours. Round to the nearest whole number.

$g_3 = 40 \qquad g_6 = 100 \qquad g_{10} = ?$

$g_6 = g_3 R^3$

$100 = 40 R^3$

$2.5 = R^3$

$\sqrt[3]{2.5} = R$

$g_{10} = g_6 R^4$

$g_{10} = 100(\sqrt[3]{2.5})^4$

$g_{10} = 100(2.5)^{4/3}$

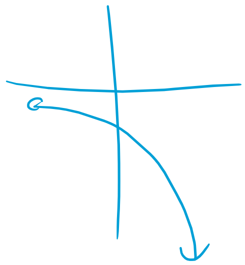
$g_{10} = 339.302$

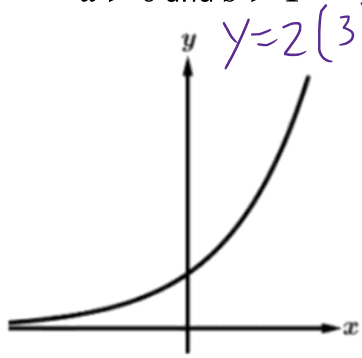
Augustus's building is getting a new roof. After 3 days of construction, 20% of the roof was complete. After 8 days of construction, 50% of the roof was complete. If progress in building the roof is best modelled by a linear function, predict how many days it will take to complete the roof.

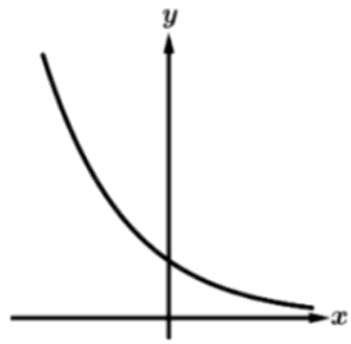
Exponential Function Graphs

$$y = a(b)^x$$

	Exponential Growth	Exponential Decay
$y = -2(3)^x$	$a > 0$ and $b > 1$ $y = 2(3)^x$	$a > 0$ and $0 < b < 1$

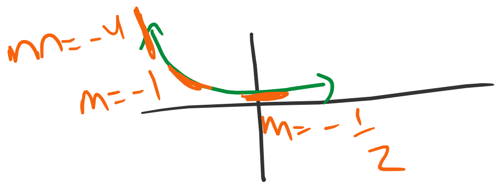






	Exponential Growth	Exponential Decay
Increasing vs Decreasing?	inc	dec
Concavity?	Concave up	up
Relative Extrema?	None	None
Points of Inflection?	None	None
End Behavior?	$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = \infty$	$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = 0$

Let $f(x) = 3\left(\frac{1}{2}\right)^x$



Growth or decay?

Is the rate of change of $f(x)$ positive or negative?

ROC negative = function is decreasing

Is the rate of change of $f(x)$ increasing or decreasing?

ROC increasing = f(x) is concave up

Describe the end behavior of $f(x)$ using limit notation.

$$\lim_{x \to -\infty} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = 0$$

Let $g(x) = -4(3)^x$

Increasing or decreasing?

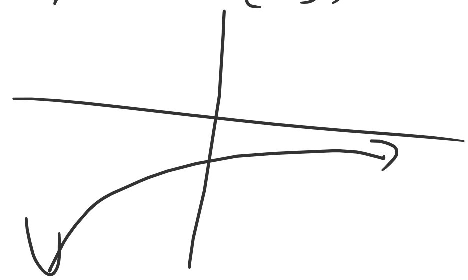
Concave up or concave down?

Describe the end behavior of $g(x)$ using limit notation.

Throughout its domain is the value of $g(x)$ positive or negative?

Let $h(x) = -4\left(\frac{2}{3}\right)^x$

$y = -4\left(\frac{2}{3}\right)^x$



$h(x)$ is...

(A) ~~Positive~~ and increasing

(B) ~~Positive~~ and decreasing

(C) Negative and increasing

(D) Negative and decreasing

Describe the end behavior of $h(x)$ as x increases without bound. Use limit notation.

$\lim_{x \to \infty} h(x) = 0$

Describe the end behavior of $h(x)$ as x decreases without bound. Use limit notation.

$\lim_{x \to -\infty} h(x) = -\infty$

Does $h(x)$ have a relative maximum? If so, find its value.

No

Let $m(x) = 4\left(\frac{2}{3}\right)^{-x}$

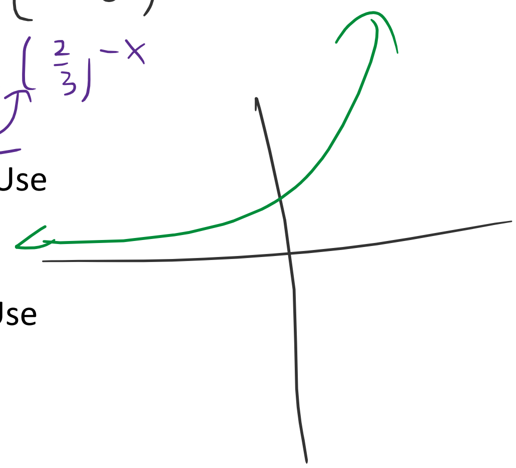
$4\left(\frac{2}{3}\right)^{-x} = 4\left(\frac{3}{2}\right)^{x}$

$4(1.5)^{x}$

$m(x)$ is...
(A) Positive and increasing
(B) Positive and decreasing
(C) Negative and increasing
(D) Negative and decreasing

$4\left(\frac{2}{3}\right)^{x}$

$4\left(\frac{2}{3}\right)^{-x}$



Describe the end behavior of $h(x)$ as x decreases without bound. Use limit notation.

$\lim_{x \to -\infty} h(x) = 0$

Describe the end behavior of $h(x)$ as x increases without bound. Use limit notation.

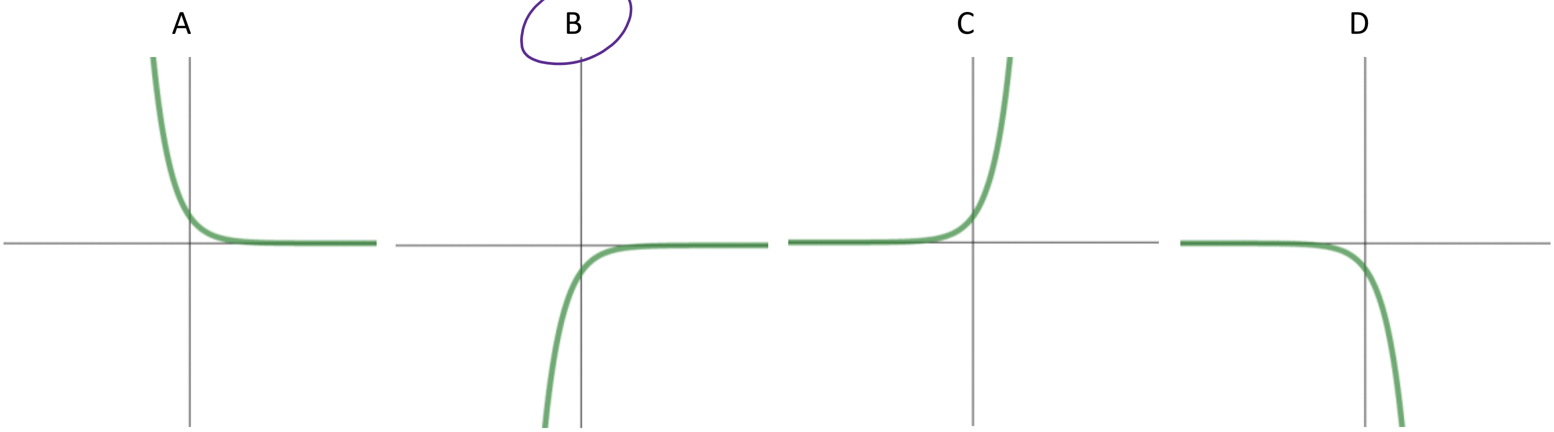
$\lim_{x \to \infty} h(x) = \infty$

Find all points of inflection of the graph of $m(x)$.

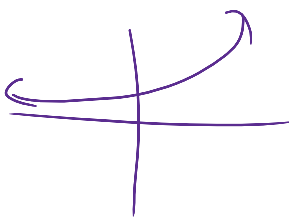
No

Let $r(x) = -\frac{2}{3}(8)^{-x}$ $= -\frac{2}{3}\left(\frac{1}{8}\right)^{x}$

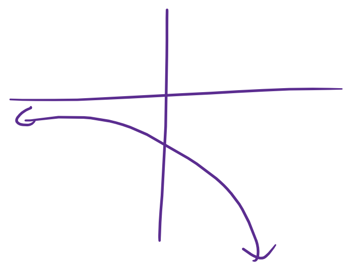
Which one is a graph of $r(x)$?



$\frac{2}{3}(8)^{x}$



$-\frac{2}{3}(8)^{x}$



$-\frac{2}{3}(8)^{-x}$